Actividad

1 Determina cuáles de las siguientes sucesiones son aritméticas. Si la sucesión es aritmética, encuentra la diferencia y el término n-ésimo para cada sucesión.

a) 2, 7, 12, 17, 22, 27, ...

b) 10, 4, -2, -8, -14, ...

c) $\frac{5}{2}, \frac{11}{6}, \frac{7}{6}, \frac{1}{2}, -\frac{1}{6}, \ldots$

d) $e^{-1}, e^{-2}, e^{-3}, e^{-4}, e^{-5}, \ldots$

e) $\frac{13}{6}, \frac{17}{12}, \frac{2}{3}, \ldots$

a = 2, 7, 12, 17, 22, 27, ...

$a_n = a_2 + (n-1)5$

b = 10, 4, -2, -8, -14, ...

$b_n = b_{10} - (n-1)6$

e = -1, -2, -3, -4, -5, ...

$e_n = e_{-1} + (n-1)1$

2 Identifica cuáles sucesiones son aritméticas. Luego escribe los cinco primeros términos de aquellas que lo sean.

a) $a_n = 4 - n$

b) $\left\{a_n = \frac{2}{n+2}\right\}$

c) $\{a_n = -n + 8\}$

d) $a_n = n + \frac{\pi}{2}$

e) $a_n = \frac{1}{2+\pi}$

f) $a_n = -\frac{2}{3}(n-1) + 2$

c = a_n = 7, 6, 5, 4, 3...

b = a = 2/1+2 = 2/3

2/2+2 = 2/4

2/3+2 = 2/5

2/4+2 = 2/6

2/5+2 = 2/7

$\ldots + 2{,}5 + 1{,}25 + 0{,}625 + 0{,}3125 + 0{,}15625\ldots$

$= 19{,}84375$

Zenon estaba equivocado porque de la suma de infinitos números puede dar un número finito.

ACTIVIDAD PÁGINA 48:

Ⓐ

$$\sum_{k=1}^{6} \frac{1}{2n} = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}$$

Ⓑ

$$\sum_{n=2}^{10} \frac{1}{n^2-1} = \frac{1}{2^2-1} + \frac{1}{3^2-1} + \frac{1}{4^2-1} + \frac{1}{5^2-1} + \frac{1}{6^2-1}$$

$$+\frac{1}{7^2-1} + \frac{1}{8^2-1} + \frac{1}{9^2-1} + \frac{1}{10^2-1}$$

$$= \frac{1}{3} + \frac{1}{8} + \frac{1}{15} + \frac{1}{24} + \frac{1}{35} + \frac{1}{48} + \frac{1}{63}$$

$$\frac{1}{80} + \frac{1}{99}$$

$$= \frac{36}{55}$$

Ⓒ

$$\sum_{n=1}^{8} (+1)^{n+1} \cdot n^2 = (1^{1+1} \cdot 1^2) + (1^{2+1} \cdot 2^2) + (1^{3+1} \cdot 3^2)$$

$$+ (1^{4+1} \cdot 4^2) + (1^{5+1} \cdot 5^2) + (1^{6+1} \cdot 6^2) + (1^{7+1} \cdot 7^2) + (1^{8+1} \cdot 8^2)$$

$$= (1 \times 1) + (1 \times 2^2) + (1 \times 3^2) + (1 \times 4^2) + (1 \times 5^2) + (1 \times 6^2) + (1 \times 7^2) + (1 \times 8^2)$$

$$= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64$$

$$= 204$$

Ⓓ

$$\sum_{n=1}^{5} 3^n(n+1) = (3^1 \cdot 1+1) + (3^2 \cdot 2+1) + (3^3 \cdot 3+1)$$

$$+ (3^4 \cdot 4+1) + (3^5 \cdot 5+1)$$

$$= 1.646$$

(e)

$$\sum_{n=1}^{9} \frac{3n-1}{n} = \frac{3\cdot 1-1}{1} + \frac{3\cdot 2-1}{2} + \frac{3\cdot 3-1}{3}$$

$$+\frac{3\cdot 4-1}{4} + \frac{3\cdot 5-1}{5} + \frac{3\cdot 6-1}{6} + \frac{3\cdot 7-1}{7}$$

$$+\frac{3\cdot 8-1}{8} + \frac{3\cdot 9-1}{9}$$

$$= 2 + \frac{6-1}{2} + \frac{9-1}{3} + \frac{12-1}{4} + \frac{15-1}{5} +$$

$$\frac{18-1}{6} + \frac{21-1}{7} + \frac{24-1}{8} + \frac{27-1}{9} +$$

$$= 2 + \frac{5}{2} + \frac{8}{3} + \frac{11}{4} + \frac{14}{5} + \frac{17}{6} + \frac{20}{7} +$$

$$\frac{23}{8} + \frac{26}{9}$$

$$= \frac{60911}{2520}$$

(f)

$$\sum_{n=1}^{5} \left(\frac{2}{7}\right)^{n-1} =$$

(g)

$$\sum_{n=1}^{10} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n\ 1}} = \left(\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{1+1}}\right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2+1}}\right)$$

$$+\left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3+1}}\right) + \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{4+1}}\right) + \left(\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{5+1}}\right)$$

$$+\left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6+1}}\right) + \left(\frac{1}{\sqrt{7}} - \frac{1}{\sqrt{7+1}}\right) + \left(\frac{1}{\sqrt{8}} - \frac{1}{\sqrt{8+1}}\right)$$

$$+\left(\frac{1}{\sqrt{9}} - \frac{1}{\sqrt{9+1}}\right) + \left(\frac{1}{\sqrt{10}} - \frac{1}{\sqrt{10+1}}\right)$$

$$= 1 - \cancel{\frac{1}{\sqrt{2}}} + \cancel{\frac{1}{\sqrt{2}}} - \cancel{\frac{1}{\sqrt{3}}} + \left(\cancel{\frac{1}{\sqrt{3}}} - \cancel{\frac{1}{2}}\right) + \cancel{\frac{1}{2}} - \cancel{\frac{1}{\sqrt{5}}} +$$

$$\cancel{\frac{1}{\sqrt{5}}} - \cancel{\frac{1}{\sqrt{6}}} + \cancel{\frac{1}{\sqrt{6}}} - \cancel{\frac{1}{\sqrt{7}}} + \left(\cancel{\frac{1}{\sqrt{7}}} - \cancel{\frac{1}{2\sqrt{2}}}\right) +$$

$$\left(\cancel{\frac{1}{2\sqrt{2}}} - \cancel{\frac{1}{3}}\right) + \cancel{\frac{1}{3}} - \cancel{\frac{1}{\sqrt{10}}} + \cancel{\frac{1}{\sqrt{10}}} - \frac{1}{\sqrt{11}}$$

$$= 1 - \frac{\sqrt{11}}{11}$$

h) $$\sum_{n=1}^{7} \left(1 + \frac{2}{n}\right)^n = \left(1 + \frac{2}{1}\right) + \left(1 + \frac{2}{2}\right) + \left(1 + \frac{2}{3}\right)$$

$$+ \left(1 + \frac{2}{3}\right) + \left(1 + \frac{2}{4}\right) + \left(1 + \frac{2}{5}\right) + \left(1 + \frac{2}{6}\right) +$$

$$\left(1 + \frac{2}{7}\right)$$

$$= 3 + (1 + 1) + \frac{5}{3} + \frac{3}{2} + \frac{7}{5} + \frac{4}{3} + \frac{9}{7}$$

$$= 3 + 2 + \frac{5}{3} + \frac{3}{2} + \frac{7}{5} + \frac{4}{3} + \frac{9}{7}$$

$$= 5 + \frac{503}{70}$$

$$= \frac{853}{70}$$

3 Halla la suma de los diez primeros términos de cada sucesión.

a) $a_n = 5^n - 5^{n-1}$

b) $a_n = \dfrac{1}{n(n+1)(n+2)}$

c) $a_n = n2^{n-1}$

d) $a_n = \left(\frac{1}{4}\right)^n + 3^{\frac{n}{5}}$

e) $a_n = 2n(2n-1)$

f) $a_n = n! - (n-1)!$

A= 4 + 20 + 100 + 2,500 + 12,500 + 62,580 + 312,500 + 1'562,500 + 7'812.500 = 9'765.624

B= 1/6 + 1/24 + 1/60 + 1/120 ... + 1/1320 = 65/240

C= 1 + 4 + 12 + 32 + 80 ... + 5120 = 9217

D= 1/4 + 3 1/3 + 1/16 + 3 3/5 ...

E= 2 + 12 + 30 + 56 ... + 380 = 1430

F= 1 + 2 + 12 + 444 ... + 131 6818944000 = 133165 7196939

4 Aplica las propiedades de la sumatoria para hallar el valor de cada suma si $a_n = 3n$, $b_n = \frac{n^3}{\ldots}$ y $c_n = \frac{1}{\ldots}$